Stochastic input model generation with Bayesian networks

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Stochastic analysis of random heterogeneous media provides useful information only if realistic input models of the material property variations are used. To represent the random fields, the K-L expansion and its variants have been used intensively. Spectral methods can be employed to represent the random coefficients in the expansion. The random coefficients in K-L expansions are uncorrelated but not, in general, statistically independent. Although the Rosenblatt transformation can capture the dependence based on conditional cumulative distribution of random variables, the estimation of joint probability from data is computationally expensive especially for high-dimensional problems. In this research, Bayesian Network (BN) dependence learning is performed to analyze the dependence structure of these random coefficients. The purpose is to explore the sparsity of dependence since a random variable may only depend on a small subset of variables rather than all the others. In this framework, the structure of BN is a directed acyclic graph composed of nodes representing the random coefficients and edges manifesting dependence between the nodes by connecting them. Starting from an initial guess of the graphical model, conditional independence (CI) tests are made to measure the dependence between nodes conditioned on another set of nodes. This is followed by the removal of edges related to independence. Finally, we decompose the network into substructures consisting of relatively small number of random variables. Then, conventional method, e.g. the inverse Rosenblatt transformation, can estimate these substructures with reduced dimensionalities.

The proposed method can construct more accurate stochastic input model by taking account of dependence between random coefficients of K-L expansion with proper computational cost. In fact, for the simplicity of computation, such random coefficients are often assumed to be independent, which is not the case for many non-Gaussian random fields. While relaxing this assumption is not difficult in theory, existing methods are still limited to low dimensional problems. The proposed method learns dependence/independence structures with Bayesian networks, which enables efficient exploration of conditional distributions based on which we map the random coefficients to \(i.i.d\). standard random variables.