A CONTINUUM SENSITIVITY ANALYSIS OF LARGE DEFORMATIONS WITH APPLICATIONS TO METAL FORMING DESIGN

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B-exam presentation
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Prof. Thomas Coleman (CS)

Support : WPAFB, AFOSR, CTC
OUTLINE OF THE PRESENTATION

- Objectives of this project
- Finite thermo-inelastic deformation analysis
- An example to motivate the need for metal forming design
- Definition and computation of sensitivity fields
- The sensitivity deformation problem
- Multistage sensitivity analysis
- Metal forming optimization examples
- Suggestions for future work
Efficient manufacture of desired shape with desired material properties in the final product

Metal forming:

MOTIVATION AND OBJECTIVES

Initial product

Plane strain forging

Final product
MOTIVATION AND OBJECTIVES

- Features of forming processes
  - Large deformation plasticity
  - Deformation induced microstructure evolution
  - Time varying contact and friction conditions
  - Thermal effects: result of mechanical dissipation
  - Damage accumulation leading to material rupture
- Mechanisms coupled in a highly non-linear fashion
- Predict the response of each of these mechanisms to variations in control variables - Sensitivity fields
MOTIVATION AND OBJECTIVES

Forming design objectives:
- Minimize energy required to deform workpiece
- Desired microstructure in the final product
- Desired final shape of the product
- Final product

Design classification:
- Single stage design
- Multistage design

Die & Process parameters:
- Single stage design
- Multistage design

Design of sequences:
- Design of sequences
OBJECTIVE

- Sensitivity analysis provides the basis for gradient based forming design optimization
- Die and process parameter design - Parameter sensitivity
- Preform design - Shape sensitivity
- Multistage design - Shape & Parameter sensitivity

Develop a deformation process design methodology
* Accurate description of the mechanics of deformation
* Efficient and accurate computation of design derivatives
HOT FORMING ANALYSIS

- Nature of coupling between deformation and thermal fields
- Features of the direct thermomechanical simulator
  - Versatile large object oriented program developed
  - Various constitutive models, constitutive integration schemes, contact and friction models, finite element types, remeshing schemes, forming applications
- Diffpack C++ library provided the basic FEM environment
DEFORMATION PROBLEM

- Principle of virtual work: UL formulation

\[ \nabla_n \cdot P_r + f_r = 0 \]

\[ \int_{B_n} P_r \cdot \nabla u \, dV_n = \int_{\Gamma} \lambda \cdot \tilde{u} \, dA_n + \int_{B_n} f_r \cdot \tilde{u} \, dV_n \]

- Newton-Raphson Method with Line Search
DEFORMATION PROBLEM

- Algorithmic division of the deformation problem

  - **Kinematic subproblem** - Given displacements
    Compute deformation gradient and strains

  - **Constitutive subproblem** - Given deformation gradient
    Update stresses, state variable & damage parameters

  - **Contact & friction subproblem** - Given location of die
    Update regions of contact, stick, slip & tractions

  - **Remeshing & data transfer subproblem** - Given old mesh
    Compute new mesh & transfer deformation fields
CONSTITUTIVE SUBPROBLEM

The following model is proposed: \( F = F_e F_p F^\theta \)
CONSTITUTIVE SUBPROBLEM

- Thermal Expansion: \( \dot{F}^{\theta} F^{\theta -1} = \beta \dot{\theta} I \)

- Inelastic response:
  - Flow rule:
    \[ D^p = \text{sym} \left( L^p \right) = \dot{F}^p F^p = \dot{\gamma} \frac{d}{dT} \Phi \]
  - \( \Phi \) is the viscoplastic potential (Gurson et al.)
  - Internal variable evolution (Anand et al.)
  - Void fraction (damage) evolution

- Hyperelastic constitutive law

- Mechanical dissipation
CONSTITUTIVE TIME INTEGRATION
CONTACT AND FRICTION SUBPROBLEM

\[ g(\mathbf{x}_{n+1}) \leq 0 \]
\[ \lambda_N = \mathbf{v} \cdot \lambda \]
\[ \lambda_N g(\mathbf{x}_{n+1}) = 0 \]

\[ \lambda_T = -\lambda + \lambda_N \mathbf{v} \]
\[ \Upsilon := \|\lambda_T\| - \mu \lambda_N \leq 0 \]
\[ \mathbf{v}_T = \chi \frac{\lambda_T}{\|\lambda_T\|} \]
\[ \chi \geq 0 \]
\[ \chi \Upsilon = 0 \]

Simo & Laursen (1992)
CONTACT AND FRICTION SUBPROBLEM

- Normal contact - penetration function
- Tangential contact - Coulomb friction model
- Augmented Lagrangian formulation used to enforce normal contact and frictional (stick) conditions
- Accurate enforcement of constraints with modest penalties
- Contact tractions given by converged Lagrange multiplier estimates
REMESHING AND DATA TRANSFER PROCEDURE

1. Compute the equilibrated body configuration $B_{n+1}^{(old)}$ corresponding to the mesh discretization $M^{(odd)}$.
   
   $\mathbf{u}_{n+1}^{(old)} = \mathbf{x}_{n+1}^{(old)} - \mathbf{x}_n^{(old)}$

2. Perform a remeshing operation on the spatial configuration $B_{n+1}$ to yield a mesh discretization $M^{(new)}$.
   
   $\mathbf{u}_{n+1}^{(new)} = \mathbf{T}_1 \left[ \mathbf{u}_{n+1}^{(old)} \right]$
   
   $\mathbf{x}_n^{(new)} = \mathbf{x}_n^{(new)} - \mathbf{u}_{n+1}^{(new)}$

3. $Q_n$ represents the necessary set of field variables that characterizes the history of the material deformation at time $t_n$.
   
   $Q_n^{(new)} = \mathbf{T}_2 \left[ Q_n^{(odd)} \right]$
   
   $Q^1 = (F^e, s)$ or $Q^2 = (F, F^p, s)$.

4. $\lambda_n = (\lambda_N, \lambda_T)$ represents the normal and tangential contact tractions at time $n$ corresponding to mesh $M^{(odd)}$.
   
   $\lambda_n^{(new)} = \mathbf{T}_3 \left[ \lambda_n^{(odd)} \right]$

5. Solve the direct deformation problem for the time increment $[t_n, t_{n+1}]$ on the mesh $M^{(new)}$. 

Without remeshing

Initial  Deformed

With remeshing
Issues involved in a robust remeshing procedure for large deformations

No remeshing

<table>
<thead>
<tr>
<th>State variable</th>
<th>Equivalent stress</th>
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With remeshing

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<td>2</td>
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</tbody>
</table>
IMPORTANCE OF METAL FORMING DESIGN

- Preform A, with damage
- Preform B, with damage
- Preform A, no damage
IMPORTANCE OF METAL FORMING DESIGN

- Damage distribution in final product

**Preform A**

**Preform B**

- Force vs stroke characteristics
SCHEMATIC OF THE SENSITIVITY ALGORITHM

- Equilibrium equation
- Design derivative of the equilibrium equation
- Material constitutive laws
  - Design derivative of the material constitutive laws
  - Time integration
- Sensitivity constitutive sub-problem
- Assumed kinematics
  - Design derivative of the assumed kinematics
- Time and space discretized weak form
- Input
- Modify
- Time and space discretized modified weak form
- Input
- Sensitivity contact sub-problem
- Regularized design derivative of the contact and friction constraints
- Contact and friction constraints
- Time integration
SHAPE SENSITIVITY OF THE DEFORMATION

Gateaux differential

\[
\Phi = \Phi(Y, t; \beta_s + \Delta \beta_s) - \Phi(Y, t; \beta_s) + O(||\Delta \beta_s||^2)
\]

\[
\dot{\Phi} = \frac{d}{d\lambda} \Phi(Y, t; \beta_s + \lambda \Delta \beta_s) \bigg|_{\lambda=0}
\]
SENSITIVITY DEFORMATION PROBLEM

- Derive a weak form for the shape sensitivity of the equilibrium equation

- Primary unknown of the weak form
  \( \dot{x} \) - sensitivity of the deformed configuration

- TL weak form

\[
\nabla_{o} \cdot P + f = 0
\]

\[
\int_{B_{o}} P \cdot \nabla_{o} \tilde{\eta} dV_{o} - \int_{B_{o}} P \left( \nabla_{o} \cdot L_{R}^{T} \right) \cdot \tilde{\eta} dV_{o} - \int_{B_{o}} \left( P L_{R}^{T} \cdot \nabla_{o} \tilde{\eta} \right) dV_{o} = \int_{\partial B_{o}} \left\{ \lambda - \left[ L_{R} \cdot (N \otimes N) \right] \lambda \right\} \cdot \tilde{\eta} dA_{o}
\]

\[
\dot{F} = \nabla_{o} \dot{x} = \nabla_{o} \ddot{x} - F L_{R}
\]

\[
\dot{P} = A \left[ \dot{F} \right] + B
\]
SENSITIVITY DEFORMATION PROBLEM

Algorithmic division of the sensitivity deformation problem

- **Kinematic subproblem** Given the deformed configuration,
  Relation between the sensitivity of (deformation gradient & displacement) etc.

- **Constitutive subproblem** Given the deformation history,
  Relation between the sensitivity of (stress, state & deformation gradient)

- **Contact & friction subproblem** Given the contact regions & tractions,
  Relation between the sensitivity of (tractions & displacement)

- **Remeshing & data transfer subproblem** Given the new mesh,
  Transfer sensitivities between the old mesh and new mesh
SENSITIVITY CONSTITUTIVE SUBPROBLEM

Badrinarayanan and Zabaras (1996)
SENSITIVITY CONSTITUTIVE SUBPROBLEM

- Obtain the linear relationship between stress, state and deformation sensitivity fields

- To achieve this (Zabaras et al. 1996):
  - Obtain rate laws governing the sensitivity of inelastic variables (plastic deformation rate, state variables)
  - Sensitivity of the hyperelastic constitutive law
  - Time integration of the rate constitutive laws to compute sensitivity fields at the end of the loading increment

- Problem identical for shape and parameter sensitivity
CONTACT SHAPE SENSITIVITY

- Essential for preform design
  - Changes in the initial preform strongly influence contact history
CONTACT PARAMETER SENSITIVITY

- Essential for die design
  - *Changes in die shape are the driving force*
SENSITIVITY CONTACT SUBPROBLEM

- Regularizing assumptions for non-differentiability
  - contact sensitivity assumption
  - friction sensitivity assumption

- Differentiate the strong form of contact constraints as opposed to the time discrete traction update

- Use higher (different) penalties than those used in the contact problem to enforce sensitivity constraints

- Sensitivity stiffness contribution - implicit nature of the contact algorithm

\[ \dot{\lambda} = A \dot{x} + b \]
SENSITIVITY CONTACT SUBPROBLEM

- Sensitivity of contact tractions

\[ \dot{\lambda} = \lambda_N \nu(y) + \lambda_N \nu(y) \nu(y) - \lambda_T \tau_1(y) - \lambda_T \tau_1(y) \]

\[ \dot{\lambda}_N = \lambda_{N_n} + \epsilon_N \dot{g} (x_{n+1}) \quad \text{Normal contact} \]

\[ \dot{\xi} = \frac{1}{\epsilon_T} \dot{\lambda}_T \quad \text{Stick} \]

\[ \dot{\lambda}_T = \left( \mu \lambda_N \frac{\lambda_T}{|\lambda_T|} \right) \quad \text{Slip} \]

- Sensitivity of the gap function and inelastic slip

\[ \ddot{\xi} = \alpha \cdot \ddot{x} + b \]

\[ \alpha = \frac{\tau_1(y)}{\|\tau_1(y)\|^2 \{1 + gK(y)\}} \]

\[ b = - \left\{ \frac{g\nu(y) \cdot \ddot{y} \cdot \ddot{y} + \tau_1(y) \cdot \ddot{y}}{\|\tau_1(y)\|^2 \{1 + gK(y)\}} \right\} \]

\[ \ddot{g} = \nu(y) \cdot \left( \ddot{y} - \dot{x} \right) \]
SENSITIVITY CONTACT SUBPROBLEM

Overview of the sensitivity contact algorithm

A continuum linear scheme

1. Solve the incremental deformation problem
2. Check if contact constraints are satisfied.
3. Augment to obtain more accurate traction estimates.
4. Solve sensitivity deformation problem (use oversized penalties for contact)

Preferred approach

A discrete iterative scheme

1. Solve the incremental direct deformation problem
2. Solve the sensitivity deformation problem
3. Check if contact constraints are satisfied.
4. Solve direct and sensitivity problem using more accurate estimates of tractions and traction sensitivities
5. Post-process

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UL SENSITIVITY FORMULATION

Parameter sensitivity in the time increment \([n,n+1]\)

\[
\begin{align*}
x_n &= \ddot{x}(X, t_n; \beta_p) \\
Q_n &= \ddot{Q}(X, t_n; \beta_p)
\end{align*}
\]

\[
x = \ddot{x}(x_n, t; \beta_p)
\]

\[
x + \ddot{x} = \ddot{x}(x_n + \ddot{x}, t; \beta_p + \Delta \beta_p)
\]

\[
x_n + \ddot{x}_n = \ddot{x}(X, t_n; \beta_p + \Delta \beta_p)
\]

\[
Q_n + \ddot{Q}_n = \ddot{Q}(X, t_n; \beta_p + \Delta \beta_p)
\]
UL SENSITIVITY FORMULATION

- Shape sensitivity in the time increment \([n, n+1]\)
UL SENSITIVITY FORMULATION

- Weak form for shape and parameter sensitivities

\[ \nabla_n^0 \mathbf{P}_r + f_r^0 = 0 \]

\[ \int_{B_n} \mathbf{P}_r \cdot \nabla_n \tilde{\eta} \, dV_n - \int_{B_n} \mathbf{P}_r \left[ \nabla_n \cdot \mathbf{L}^T_n \right] \cdot \tilde{\eta} \, dV_n - \int_{B_n} \left( \mathbf{P}_r \mathbf{L}^T_n \cdot \nabla_n \tilde{\eta} \right) \, dV_n \]

\[ = \int_{\Gamma} \left\{ \lambda - \left[ \mathbf{L}_n \cdot (n \otimes n) \right] \lambda \right\} \cdot \tilde{\eta} \, dA_n \]

- Kinematic relationships

\[ \mathbf{\dot{F}}_r = \nabla_n^0 \mathbf{x} = \nabla_n \mathbf{\ddot{x}} = \mathbf{F}_r \mathbf{L}_n \]

\[ \mathbf{\ddot{F}} = \mathbf{\dot{F}}_r F_n + \mathbf{F}_r \mathbf{\ddot{F}}_n \]
PERFORMANCE OF ASSUMED STRAIN ANALYSES

- Extension of the F-bar method by Owen (96)
TREATMENT OF INCOMPRESSIBILITY

- Sensitivity of the assumed deformation gradient

\[ F_h^{ave} = \left\{ \epsilon \, F_h + (1-\epsilon) \left[ \frac{J_h}{J} \right]^\frac{1}{3} \, F_h \right\} + \]

\[ \frac{1-\epsilon}{3} \left\{ \sum_{a=1}^{NINT} J_h(\tilde{\xi}_a)tr \left[ F_h(\tilde{\xi}_a) F_h^{-1}(\tilde{\xi}_a) \right] N_a \right\} \tilde{J}^{-1}_h \tilde{F}_h - tr \left[ f_h \, \tilde{F}_h \right] \tilde{F}_h \}

- Modified sensitivity weak form (stabilized F-bar method)

\[ S_{h}^{int} = \sum_{\epsilon} \left[ \int_{\Omega_{\epsilon}} P_{\epsilon} \left( F_h^{ave} \right) \cdot \nabla_{n} \tilde{\eta}_{h} \, dV_{n} - \int_{\Omega_{\epsilon}} \left( P_{\epsilon}(F_h^{ave}) \left[ \nabla_{n} \cdot L_{h}^{T} \right] \right) \cdot \tilde{\eta}_{h} \, dV_{n} \right. \]

\[ - \int_{\Omega_{\epsilon}} \left[ P_{\epsilon}(F_h^{ave}) L_{h}^{T} \cdot \nabla_{n} \tilde{\eta}_{h} \right] \, dV_{n} \]
DATA TRANSFER FOR SENSITIVITY PROBLEM

1. Solve the direct deformation problem for the time increment $[t_n, t_{n+1}]$.

2. Use an appropriate transfer operator to compute the sensitivities of the nodes of mesh $\mathcal{M}^{(new)}$ at time $t_n$. This transfer is used in the computation of $L_n^{(new)}$ in the UL sensitivity analysis.

$$\bar{x}_n = S_1 \left[ x_n^{(odd)} \right]$$

3. Let $\bar{Q}_n$ represent the sensitivity of the necessary set of variables that characterizes the history of the material deformation sensitivity at time $t_n$. This set is known corresponding to the mesh discretization $\mathcal{M}^{(odd)}$ and one must transfer these variables to the new mesh using appropriate transfer operators

$$Q_n^{(new)} = S_2 \left[ Q_n^{(odd)} \right]$$

4. Let $\bar{\lambda}_n = (\lambda_{N_n}, \lambda_{T_n})$ represent the normal and tangential contact tractions sensitivities at time $n$ corresponding to mesh $\mathcal{M}^{(odd)}$.

$$\lambda_n = S_3 \left[ \lambda_n^{(odd)} \right]$$

5. Solve the sensitivity deformation problem for the time increment $[t_n, t_{n+1}]$ on the mesh $\mathcal{M}^{(new)}$. 

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MULTISTAGE SENSITIVITY ANALYSIS

- Design sensitivity of the current forming stage due to variations in parameters of the current forming stage

\[ x = \bar{x}(X, t; \beta_X, \beta_Y) \]

\[ X = \bar{X}(Y, t_0; \beta_Y) \]

\[ Q = \bar{Q}(Y, t_0; \beta_Y) \]

\[ F_X, F_Y \]

\[ B, B' \]

\[ \beta_X + \Delta \beta_X \]

\[ x + \delta x = \bar{x}(X, t; \beta_X + \Delta \beta_X, \beta_Y) \]
MULTISTAGE SENSITIVITY ANALYSIS

- Design sensitivity of the current forming stage due to variations in design parameters of previous forming stages

\[
\begin{align*}
X &= \bar{X}(Y, t_o; \beta_Y) \\
Q &= \bar{Q}(Y, t_o; \beta_Y) \\
\beta_Y + \Delta \beta_Y = & X + \vec{X} = \bar{X}(Y, t_o; \beta_Y + \Delta \beta_Y) \\
Q + \bar{Q} &= \bar{Q}(Y, t_o; \beta_Y + \Delta \beta_Y) \\
x &= \bar{x}(X, t; \beta_X, \beta_Y) \\
\beta_X &= \bar{F}_X \\
x + \vec{x} &= \bar{x}(X + \vec{X}, t; \beta_X, \beta_Y + \Delta \beta_Y)
\end{align*}
\]
MULTISTAGE SENSITIVITY ANALYSIS

Effect of processing history on the current forming stage:

\[
\dot{\phi} = \frac{\partial \Phi(Y, t; \partial B_o, Q)}{\partial (\partial B_o)} \left[ \frac{\partial (\partial B_o)}{\partial \beta} [\Delta \beta_Y] \right] + \sum_i \frac{\partial \Phi(Y, t; \partial B_o, Q)}{\partial Q_i} \left[ \frac{\partial Q_i}{\partial \beta} [\Delta \beta_Y] \right]
\]

Generalization to M stages:

- \( q = [1 .. M] \)

- \( \Lambda_q \) represents preform shape and properties after \( q^{th} \) stage

- \( \beta_q \) is the design space of the \( q^{th} \) stage

TL or UL sensitivity formulation within each stage
NUMERICAL EXAMPLES

Sensitivity validation using FDM as reference

Normal contact traction sensitivity

Tangential contact traction sensitivity
DESIGN SENSITIVITY VALIDATION

No remeshing

Stress sensitivity (DDM)
8  0.1400
7  0.1286
6  0.1171
5  0.1057
4  0.0943
3  0.0829
2  0.0714
1  0.0600

DDM

Stress sensitivity (FDM)
8  0.1400
7  0.1286
6  0.1171
5  0.1057
4  0.0943
3  0.0829
2  0.0714
1  0.0600

With remeshing

Stress sensitivity (DDM)
8  0.1400
7  0.1286
6  0.1171
5  0.1057
4  0.0943
3  0.0829
2  0.0714
1  0.0600

FDM

Stress sensitivity (FDM)
8  0.2500
7  0.2071
6  0.1643
5  0.1214
4  0.0786
3  0.0357
2  -0.0071
1  -0.0500
FORMING DESIGN PROBLEMS

Features of a typical optimization problem

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Constraints</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material usage</td>
<td>Press force</td>
<td>Identification of stages</td>
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<tr>
<td>Plastic work</td>
<td>Press speed</td>
<td>Number of stages</td>
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<td>Uniform deformation</td>
<td>Product quality</td>
<td>Preform shape</td>
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<tr>
<td>Microstructure</td>
<td>Geometry restrictions</td>
<td>Die shape</td>
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<tr>
<td>Desired shape</td>
<td>Cost</td>
<td>Mechanical parameters</td>
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<tr>
<td>Residual stresses</td>
<td>Processing temperature</td>
<td>Thermal parameters</td>
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</table>
PREFORM DESIGN EXAMPLE

Objective function

Minimize barreling in the final product
CLOSED-DIE PREFORM DESIGN PROBLEM

- Objective is the desired final shape

- Force

![Diagram of force vs. stroke for initial and optimal preforms]
TWO STAGE DESIGN EXAMPLE

Initial

Iteration 3

Iteration 6

Optimal

Preforming stage

Finishing stage
TWO STAGE DESIGN EXAMPLE

- Variation of equivalent plastic strain in the final product

- Variation of the internal state variable in the final product
TWO STAGE SENSITIVITY VALIDATION

Preforming stage

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<tbody>
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<td>8  0.0070</td>
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<td>2  -0.0003</td>
<td>2  -0.0033</td>
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<tr>
<td>1  -0.0020</td>
<td>1  -0.0050</td>
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<td>1  -0.0020</td>
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DDM

FDM
TWO STAGE SENSITIVITY VALIDATION

Finishing stage

State

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Stress

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DDM

FDM

Materials Process Design and Control Group
Cornell University
INDUSTRIAL DESIGN PROBLEMS

- Two stage design for engine disk forging
FUTURE RESEARCH AND OPEN ISSUES

- Thermo-mechanical design
- Use ideal forming methods for design of sequences
- Adaptive analysis driven by direct & sensitivity error indicators
- Explicit microstructure optimization models
- More complex forging geometries and design features
- Transition from academia to industry