Predictive modeling of physical processes in heterogeneous materials requires innovations in mathematical and computational thinking. While recent multiscale approaches have been successful in modeling the effects of microstructure to macroscopic response, a significant grand challenge remains in understanding the effects of microstructural and other uncertainties in characterization of properties and in the design of heterogeneous materials. To address these problems, we are developing a mathematical framework that provides a paradigm shift in the predictive modeling of complex systems in the presence of uncertainties in order to address two major limitations in modeling stochastic PDEs: (I) The stochastic inputs are mostly based on ad hoc models, and (2) The number of independent stochastic parameters is typically very high. To address the former, we are developing non-linear data-driven model reduction strategies to utilize experimentally available information based on low-order realistic models of input uncertainties. To address the latter, we are developing a low-complexity surrogate model of the high-dimensional stochastic multiscale system under consideration. A number of examples will be discussed in the data-driven representation of random heterogeneous media and in modeling physical processes (deformation, thermal/hydrodynamic transport, etc.) in such media.